RATES OF CHANGE

1. A cuboid has a square base with sides of p cm each and its height is 4p cm. If p increases at a rate of 0.03 cm s^{-1}, find the rate of change in volume when its surface area is 288 cm^2.

2. The volume, V cm^3, of a sphere with radius r cm is given by $V = \frac{4}{3}\pi r^3$. If the radius of a sphere increases at a constant rate of 2 cm/s, find the rate at which its volume increases when its radius is 8 cm.

3. The surface are, L cm^2, of a solid is related to its temperature, θ°C, by the equation $L = \frac{\theta^2}{12} + 8$ If the temperature of solid increases at a rate of 0.4°Cs^{-1}, find the rate at which its surface area increases when its temperature is 30°C.

4. The volume of a balloon, in the shape of a sphere, decreases at a rate of 4 cm^{3}s^{-1} due to leakage. Find the rate of change in the radius when it is 3 cm.

5. The area of a piece of square metal sheet decreases at a rate of $3\text{cm}^2\text{s}^{-1}$ when it is cooled. Find the rate of change in the length of its side when its side is 5 cm.

6. The pressure P and the volume V of a gas is related by the equation PV=900. If P increases at a rate of 4 units per minute, find the rate at which V changes when P=50 units.

7. The surface area of a cube with sides of x cm each is increasing at a rate of $10\text{cm}^2\text{s}^{-1}$.
   a) Find the rate at which the length of each side increases when each side is 5 cm long.
   b) Hence, find the rate at which its volume increases at the same moment.

8. A circle with a radius of r cm has a circumference of P cm and an area of A cm^2.
   a) Show that $\frac{dA}{dP} = r$.
   b) Given that the area of the circle increases at a rate of $2\text{cm}^2\text{s}^{-1}$, find the rate at which its circumference increases when its radius is 6 cm.

9. A sphere with a radius of r cm has a surface area of A cm^2 and the volume of V cm^3.
   a) Find $\frac{dV}{dA}$ in terms of r.
   b) If the surface area of the sphere increases at a rate of 0.5π cm^2s^{-1} when its surface area is 64π cm^2, find the rate at which its volume increases at the same moment.

10.
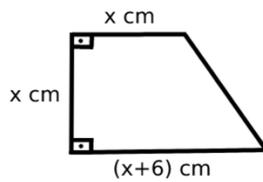
The diagram shows a trapezium.

    a)   Calculate its area in terms of x.

    b)   If x increases at a rate of 0.5 cm s^{-1}, find the rate at which the area of the trapezium increases when x=4.

11.   In a study to determine the relationship between an object and its image produced by a convex lens, it is found that the object distance, x cm, and its image distance, y cm, is related by the equation $\frac{1}{y} = \frac{1}{20} - \frac{1}{x}$.

    a)   Find $\frac{dy}{dx}$ in terms of x.

    b)   If the object distance increases at a rate of 3 cm s^{-1}, find the rate at which its image distance increases when the object distance is 50 cm.

ANSWER KEY

1. $5.76 \text{ cm}^3\text{s}^{-1}$

2. $512\pi \text{ cm}^3\text{s}^{-1}$

3. $2\text{cm}^2\text{s}^{-1}$

4. $-\frac{1}{9\pi} \text{ cm/s}$

5. -0.3 cm/s

6. -36/25 units per minute

7. a)1/6 cm/s b)25/2 cm^3s^{-1}

8. b) 1/3 cm/s

9. a) r/2 b)$\pi \text{ cm}^3\text{s}^{-1}$

10. a) $(x^2 + 3x) \text{ cm}^2$ b)5.5 cm^2s^{-1}

11. a)$-\frac{400}{(x-20)^2}$ b)-4/3 cm/s