

QUADRATIC FUNCTIONS

CANSU OLCE

A STAR MATHS (www.astarmaths.com.au)

1. Following quadratic functions in the form $f(x) = ax^2 + bx + c$, state the value of a, and c.
- $f(x) = 2x - 5 - 3x^2$
 - $f(x) = 5 - (2 - x)(1 + x)$

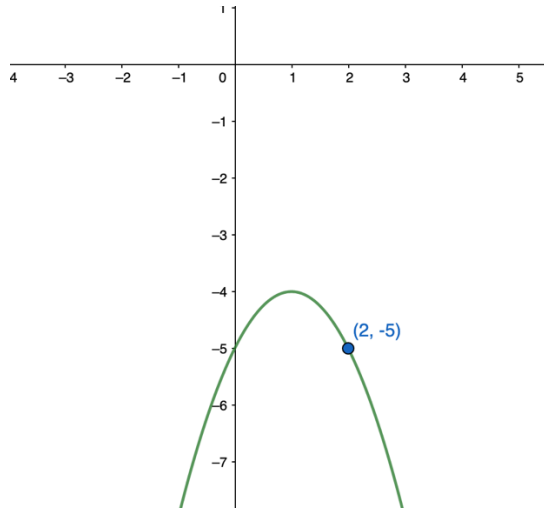
2. For each quadratic function below; state whether the graph curves upward or downward.
- $f(x) = (1 - x)(3 + 2x)$
 - $f(x) = 2 - 3x(1 - x)$

3. For each quadratic function below, determine whether the graph intersects the x-axis at two points, touches the x-axis at only one point or does not intersect the x-axis.
- $f(x) = 12x - 9 - 4x^2$
 - $f(x) = 3x(x - 2) - 5$
 - $f(x) = 7 - 2x(5 - 3x)$

4. Find the maximum or minimum points of the graph of each quadratic function below by completing squares.
- $f(x) = 2x^2 - 4x + 11$
 - $f(x) = 1x + 5 - 3x^2$

5. Sketching the graph of each of the following quadratic functions.
- $f(x) = 2(x + 1)^2 - 7$
 - $f(x) = 4 - 3(x - 2)^2$
 - $f(x) = 4[(x + 3)^2 + 2]$
6. The quadratic function $f(x) = 2x^2 + px + q$ has a minimum value of -18 when $x=1$.
- Find the values of p and q .
 - With the values of p and q from (a), find the values of x where the graph of $f(x)$ intersects the x -axis.
 - Hence, sketch the graph of $f(x)$, indicating the minimum point and x -intercept on the sketch.
7. Find the range of values of x which satisfies each inequality below.
- $x^2 - x \geq 6$
 - $x(5 - 2x) \geq 2$
8. Given the quadratic equation $2x^2 + (m + 2)x - (m - 4) = 0$ has two distinct roots. Find the range of values of m .
9. The function $f(x) = x^2 - 2px + 2p^2$ has a minimum value of $q^2 + 2p$, where p and q are constants.
- Using the method of completing the square, show that $q=p-1$.
 - Hence, or otherwise, find the values of p and q if the graph of the function is symmetrical about $x = q^2 - 1$.

10.



The diagram shows the graph of a function $y = -(x - m)^2 - 4$, where m is a constant. Find
a) the value of m .
b) the equation of the axis of symmetry.
c) the coordinates of the maximum point.

11. Find the range of values of x for which $x(x - 3) \leq 3$

12. Find the possible values of k if the graph of the quadratic function $f(x) = px^2 + 8x + 3$, where p is a constant, does not intersect the x -axis. Find the range of values of p .

13. Find the possible values of k if the graph of the quadratic function $f(x) = 4x^2 - 3kx + 2k + 1$, where k is a constant, touches the x -axis at only one point.

14. Given the graph of the quadratic function $f(x) = 3x^2 - 6x - m$, where m is a constant, intersects the x -axis at two different points. Find the range of values of m .
15. Given the graph of the quadratic function $f(x) = 2x^2 - 5x - k$, where k is a constant, intersects the x -axis at two different points. Show that $k > -25/8$.
16. Given the graph of the quadratic function $y = 2x + c - 7 - 3x^2$, where c is a constant, does not intersect the x -axis. Show that $c < 20/3$.
17. Find the range of values of m such that the graph of the quadratic function $y = 2x^2 - 5x - 1 - 2m$, where m is a constant, intersects the x -axis at two different points.
18. The graph of the quadratic function $f(x) = m(x + a)^2 + b$, where m , a and b are constants, has a minimum point $(-1, 8)$ and passes through point $(0, 15)$. Find the values of a , b and m .
19. The graph of the quadratic function $f(x) = r(x + p)^2 + q$, where r , p and q are constants, has a maximum point $(2, 3)$ and passes through point $(1, 0)$. Find the values of p , q and r .

20. Find the range of values of p if the quadratic function $f(x) = x^2 - 6x + p - 2$, where p is a constant, is always positive for all values of x .

21. Find the range of values of c if the quadratic function $f(x) = 2x - x^2 + c + 5$, where c is a constant, is always negative for all values of x .

22. A quadratic function is given as $f(x) = 2x^2 - 3x - 2$.

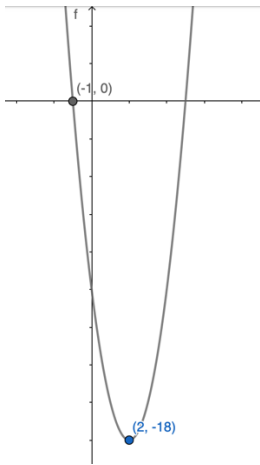
- Find the values of x where the graph of function $f(x)$ intersects the x -axis.
- Find the maximum or minimum of the graph.
- Sketch the graph of function $f(x)$.

23. Given the function $f(x) = q + px - x^2$, where p and q are constants, has a maximum point $(1, 4)$.

- Find the values of p and q .
- With the values of p and q from (a), find the values of x where the graph of function $f(x)$ intersects the x -axis.
- Sketch the graph of function $f(x)$.

24. A quadratic function is given as $y = 2x^2 - 4x + 5$.
- Show that the graph of the function does not intersect the x-axis for all values of x.
 - Find the maximum or minimum point of the graph by completing the square.
 - Find the values of y where the graph intersects the y-axis.
 - Sketch the graph of the function.

25.



The diagram shows the graph of a quadratic function $f(x) = p(x + m)^2 + n$, where m, n are p are constants. The graph of $f(x)$ has a minimum point $(2, -18)$ and intersects the x-axis at point $(-1, 0)$.

- Find the values of m, n and p.
 - If the graph is reflected in the x-axis, write the equation of the resulting graph.
26. By finding the values of x where a graph intersects the x-axis, sketch the graph of the quadratic function $y = x(x - 5)$. Hence, find the range of values of x where $x(x-5) < 0$.

27. By finding the values of x where a graph intersects the x -axis, sketch the graph of the quadratic function $y = (2x + 5)(x - 2)$. Hence, find the range of values of x where $(2x + 5)(x - 2) > 0$.
28. Given that $f(x) = x(5 - x)$, find the range of values of x such that $f(x) > 4$.
29. Given that $f(x) = (3x - 2)(x + 3)$, find the range of values of x such that $f(x) < 2 - 3x$.
30. A quadratic function is given as $y = (2 - m)x^2 - 8x + 3$, where m is a constant. Find the range of values of m such that the graph of the function has a minimum point and intersects the x -axis at two different points.
31. Rewrite the expression $-2x^2 - 6x + 5$ in the form $p(x + q)^2 + r$, where p , q and r are constants. Hence, state the maximum and minimum value of the function $f(x) = -2x^2 - 6x + 5$ and the corresponding value of x .

32. Find the maximum value of the function $f(x) = 1 + 3x - 2x^2$ by using the method of completing square. Hence, state the equation of the axis of symmetry of the graph of $f(x)$.

33. Find the range of values of x for which $3(2x^2 - x) < 2x + 1$.

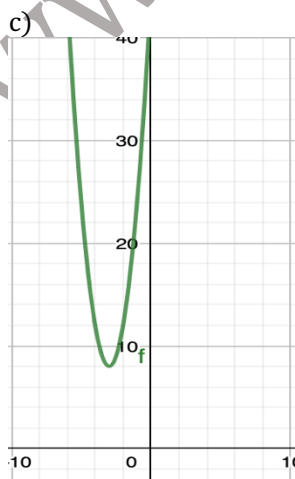
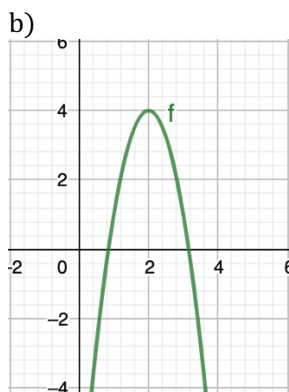
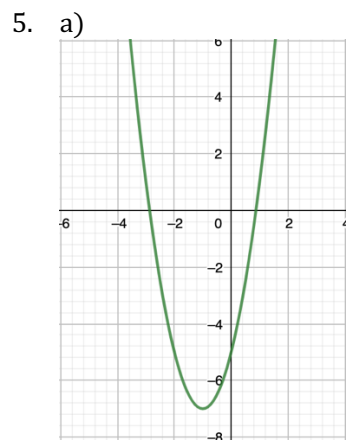
34. Find the range of values of x for which $x(x - 5) \leq 14$.

35. Find the range of values of x for which $(x + 2)(x - 3) > 3x - 1$

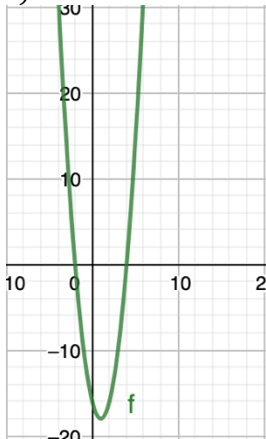
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ANSWER KEY

1. a) $a=-3, b=-5$
b) $a=1, b=3$
2. a) downwards
b) upwards
3. a) the graph touches the x-axis at only one point.
b) the graph intersects the x-axis at two points.
c) the graph does not intersect the x-axis.
4. a) the minimum point $(1,9)$
b) the maximum point $(2,17)$



6. a) $p=-4$ and $q=-16$
 b) $x=-2, x=4$
 c)



7. a) $x \leq 2$ or $x \geq 3$
 b) $1/2 \leq x \leq 2$

8. $m < -14$ or $m > 2$
 9. When $p=0, q=-1$
 When $p=3, q=2$

10. a) $m=1$
 b) $x=1$
 c) $(1, -4)$

11. $-1 \leq x \leq 4$

12. $p > 16/3$

13. $k = -4/9$ or 4

14. $m > -3$

15.

16.

17. $m > -33/16$

18. $a=1, b=8, m=7$

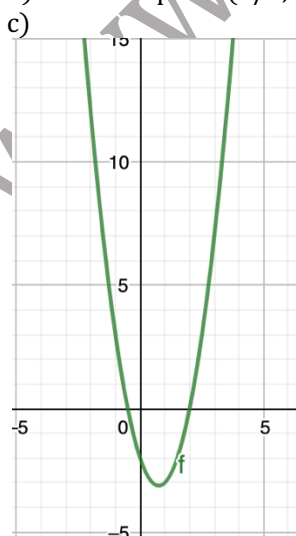
19. $p=-2, q=3, r=-3$

20. $p > 11$

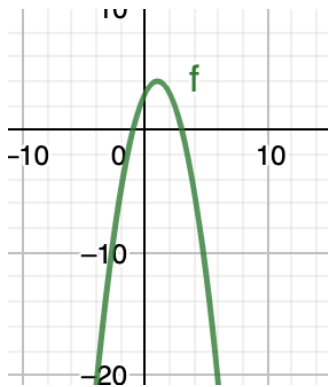
21. $c < -6$

22. a) $x = -1/2, 2$

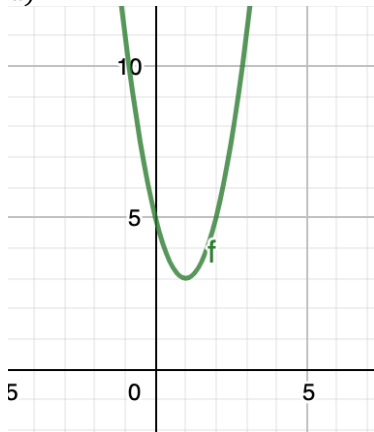
b) minimum point $(3/4, -25/8)$



23. a) $p=2, q=3$
 b) $x=-1, 3$
 c)



24. b) minimum point (1,3)
 c) $y=5$
 d)



25. a) $m=-2, n=-18, p=2$
 b) $f(x) = -2(x - 2)^2 + 18$

26. $0 < x < 5$

27. $x < -5/2$ or $x > 2$

28. $1 < x < 4$

29. $-4 < x < 2/3$

30. $-10/3 < m < 2$

31. $-2\left(x + \frac{3}{2}\right)^2 + \frac{19}{12}$, max value $19/12$, $x = -3/2$

32. Maximum value $17/8$; axis of symmetry $x = 3/4$

33. $-1/6 < x < 1$

34. $-2 \leq x \leq 7$

35. $x < -1$ or $x > 5$