OPTIMIZATION

CANSU OLCE

A STAR MATHS (www.astarmaths.com.au)

1. Find the turning points of the curve $y = x^3 - x^2 - x + 4$. Hence, determine whether each turning point is a maximum or minimum point.

2. Find the turning points of the curve $y = \frac{1}{2x} + 2x$. Hence, determine whether each turning point is a maximum or minimum.

3. Find the turning point of the curve $y = 2x + \left(3 - \frac{x}{2}\right)^2$. Hence, determine the maximum or minimum value of y.

4. The curve $y = 2x^3 + bx^2 + cx$, where b and c are constants, has a turning point at (1,5).
   a) Find the values of b and c.
   b) Determine whether the turning point at (1,5) is a maximum or minimum point.
   c) Find another turning point of the curve and determine whether it is a maximum or minimum point.

5. A curve has the function $y = 9 + 6x - x^2$.
   a) Find the value of x when y has a maximum value.
   b) Hence, find the maximum value of y.

6. A curve has the function $y = x^2 + \frac{16}{x}$.

a) Find the value of x when y has a minimum value.

b) Hence, find the minimum value of y.

7. Given a function $S = \frac{r^2}{4} - 3r + 12$.

a) Find the value of r such that S is minimum.

b) Hence, find the minimum value of S.

8. Given $2x + y = 30$ and $L = \frac{1}{2}xy$.

a) Find the value of x such that L is maximum.

b) Hence, find the maximum value of L.

9. Given $p + q = 12$, where $p > 0$ and q$>$ 0. Find the maximum value of p^2q.

10. A closed rectangular box has a square base with sides of p cm each and a height of q cm. the volume of the box is 64 cm^3.

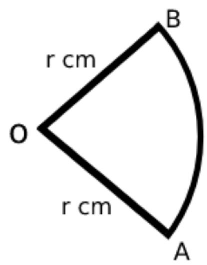
a) Show that the total surface area, L cm^2, of the box given by $L = 2p^2 + \frac{256}{p}$.

b) Find the minimum possible value of the total surface area of the box.

11. A closed cylinder with a radius of r cm and a height of h cm is constructed in such a way that its total surface area is $150\pi\ cm^2$.

a) Show that the volume of the cylinder is $\pi r(75 - r^2)\ cm^2$.

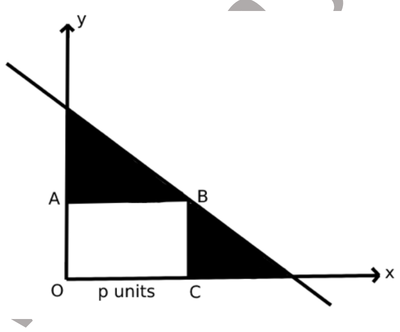
b) Hence, find the maximum possible volume of the cylinder.

12. 

The diagram shows a 50 cm wire which has been bent to form the shape of a sector with centre O and a radius of r cm.

a) Show that the area enclosed by the sector is r(25-r) cm^2

b) Hence, find the maximum area enclosed by the sector.

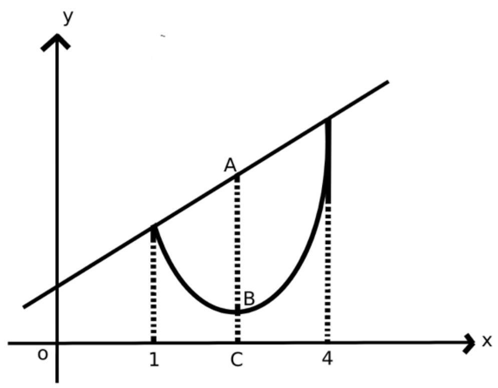
13. 

In the diagram, OABC is a rectangle with point B lying on the straight line $\frac{x}{8} + \frac{y}{4} = 1$.

a) Show that the area of the shaded region is $\left(16 - 4p + \frac{p^2}{2}\right) unit^2$.

b) If p is allowed to vary such that $0 \leq p \leq 8$, find the minimum possible area of the shaded region.

14.



The diagram shows the straight line $y = x + 1$ intersecting with the curve $y = x^2 - 4x + 5$ at the points where $x = 1$ and $x = 4$. C is a point with coordinates (p,0) such that 1<p<4. ABC is a straight line parallel to the y-axis.

a) Find the length of AB in terms of p.

b) Hence, find the value of p such that the length of AB is a maximum.

15. Find the coordinates of the turning point of the curve $y = (2 - x)^2$.

16. Find the coordinates of the two turning points of the curve $y = x(x^2 - 3)$.

17. Find the x coordinates of the two turning points of the curve $y = 9x + \frac{4}{x}$.

18. Find the coordinates of the turning point of the curve $y = 8x + \frac{4}{2x^2}$.

19. Find the x coordinates of the turning point of the curve $y = \frac{16x^3+4x^2-1}{2x^2}$.

20. Find the x coordinates of the turning point of the curve $y = \frac{3}{x^2-2x}$.

21. Find the coordinates of the maximum point of the curve $y = x + \frac{4}{x}$.

22. The curve $y = \frac{x^3+16}{4x}$ has a stationary point.

  a) Find the x-coordinate of the turning point.

  b) Determine whether the stationary point is a maximum or a minimum point.

23. Given that $V = 10r - \frac{5}{6}r^3$, find the positive value of r that makes V a maximum.

24. Find the minimum value of $A = x^2 + y^2$ if $x + y = 10$.

25. Two quantities, u and v, vary in such a way that $v - u^2 = 9$. Another quantity P is defined by $P = \frac{v}{u}$. Find the positive value of u which makes P a minimum.

26. Find the minimum value of $L = x + y$ if x and y are related by the equation $xy = 25$ if x>0 and y>0.

27. Two positive variables, x and y, vary such that $x^3 + y = 3$. Given that $z = 4x^2 + y$, find the value of x such that z is a maximum.

28. Given that $xy = 5$ and $w = x + 5y$, find the value of x and of y such that w is a minimum.

29. Given that $p + q = 4$ and $A = 2p^2 + q^2$, find the minimum value of A.

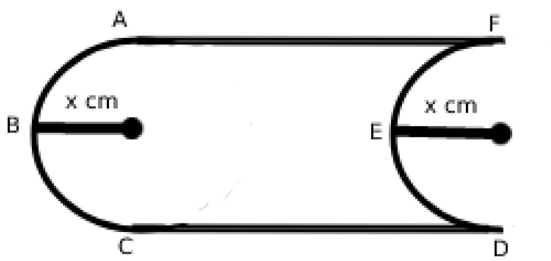
30. Find the equation of the two normal at the intersection points of the curve $y = 3 + 2x - x^2$ with the straight line $y = -5$.

31. The gradient of the curve $y = \frac{p}{qx+2}, x \neq -\frac{2}{q}$ at the point (-2, -1) is -3/4. Find the possible value of p and q such that $p \neq 0$.

32. The tangent to the curve $y = ax^2 + bx + c$ at the point where $x = -1$ is parallel to the straight line $8x + y - 2 = 0$. Given that (1, 4) is a turning point on the curve, find the value of a, of b and of c.

33. The curve $y = 2x^2 + qx$ has a turning point at (p,-2). Find the values of p and the corresponding values of q.

34.



Zara is given a piece of wire to construct an enclosed region, as shown in the diagram. ABC and FED are the arcs of two semicircles, both of radius x cm. The area of the enclosed region ABCDEF is $1000cm^2$.

a) Show that the length of the wire needed, P cm, is given by $P = 2\pi x + \frac{1000}{x}$.

b) Taking $\pi = 3.142$, calculate the value of x that makes p a minimum and show that the value of P is a minimum.

35.

A solid is to be constructed in the shape of a cylinder of radius x m and length y m, with two hemispheres attached, one to each end of the cylinder, as shown in the diagram.

a) If the total surface area of the solid is A m^2, express A in terms of x and y.

b) If the volume of the whole solid is $\frac{\pi}{6}m^3$, show that $y = \frac{1-8x^3}{6x^2}$.

c) Hence, show that $A = \frac{4}{3}\pi x^2 + \frac{\pi}{3x}$.

d) Show that the minimum total surface area of the solid is $\pi\ m^2$.

36. A closed cylindrical water tank, with a radius of r cm, is to be constructed using aluminium sheets to hold $432\pi\ m^3$ of water.

a) If the total surface area of the water tank is A m^2, show that A is given by $A = 2\pi r^2 + \frac{864\pi}{r}$.

b) The cost of aluminium sheets is \$12 per m^2. Calculate the minimum cost required.

ANSWER KEY

1. (-1/3, 113/27) maximum and (1, 3) minimum point
2. (-1/2, -2) maximum and (1/2, 2) minimum point
3. (2, 8) minimum value of y=8.
4. a)b=-9 c=12

   b)maximum point

   c)(2, 4) minimum point
5. a)x=3

   b)18
6. a)x=2

   b)12
7. a)r=6

   b)3
8. a)15/2

   b)56.25
9. 256
10. b)96cm^2
11. 250πcm^3
12. 156.25
13. 8$unit^2$
14. a)$5p - p^2 - 4$

    b)p=5/2
15. (2, 0)
16. (1, -2), (-1, 2)
17. $\pm$2/3
18. (1/2, 6)
19. x=-1/2
20. (1, -3)
21. (-2, -4)
22. a)x=2

    b)minimum
23. 2
24. 50
25. u=3
26. 10
27. 8/3
28. x=5, y=1
29. 32/3

30. 6y=-x-32
31. p=4, q=3
32. a=2, b=-4, c=6
33. p=1, q=-4 or p=-1 q=4
34. b)x=12.616
35. a) A=4πx^2+2 πxy
36. b) $8143.01