CHALLENGING LOGARITHM QUESTIONS

CANSU OLCE

A STAR MATHS (www.astarmaths.com.au)

1. $5^{x-3} = 2$

   Find x.

2. $\log_3[1 + \log_2(x-1)] = 1$

   $x = ?$

3. $\log_5(\log_2(x+1)) = 1$

   $x = ?$

4. $\log_{20} 16 + \log_{20} 25 = ?$

5. $\log_5 2 = a$

   $\log_5 3 = b$

   Find $\log_5 162$ in terms of a and b.

6. $If\ \log 2 = 0.301$, then find the approximate value of $\log 40$.

7. $\log_5 49 \times \log_7 125 = ?$

8. $\dfrac{1}{\log_5 20} + \dfrac{1}{\log_4 20} = ?$

9. $\log_x y = 3$

   $\log_{xy} y^2 = ?$

10. $\log_2 3 = a$

    Find $\log_{12} 6$ in terms of a.

11. $e^{2+ln4} = ?$

12. $3^{2+\log_{\sqrt{3}} 2} = ?$

13. $\log_2 a = m$

    Find $\log_2 a^3$ in terms of m.

14. $Sketch\ f(x) = \log_4(x-2)$

15. $\log_4(4x-8)=2$

$\log_x 6 =?$

16. $\log_8(2^{x+3}\times 4^x)=3$

Find x.

17. $\ln x+\ln 2=1$

$x =?$

18. $\log_3 a+\log_{27} a+\log_9 a=\frac{11}{3}$

a=?

19. $\ln(x+1)=1+\ln(x-1)$

Find x.

20. $(\log_2 x)^2-\log_2 x=2$

Find the sum of possible x values.

21. $\log_2(x-6)\le 2$

Find x.

22. $\log 5=a$

Find $\log 4$ in terms of a.

23. $x^{lnx}-e^{12}x=0$

Find possible x values.

24. $\log_4 9\cdot\log_{\sqrt{3}}125\cdot\log_{25}8=?$

25. $3^{\log_2 x}=7^{\log_2 3}\Rightarrow x=?$

26. $\ln(ab)=5$

$\ln\left(\frac{a}{b}\right)=1$

$a=?$

27. $3\sqrt{\ln x}-2\ln\sqrt{x}=0$

Find the possible x values.

28. A group of ten leopards is introduced into a game park. After t years the number of leopards, N, is modelled by $N = 10\ e^{0.4t}$.

(a) How many leopards are there after 2 years?

(b) How long will it take for the number of leopards to reach 100?

(c) Find the initial number of leopards.

(d) How long will it take for the number of leopards to double?

29. The mass m kg of a radio-active substance at time t hours is given by $m = 4e^{-0.2t}$.

(a) Write down the initial mass.
(b) The mass is reduced to 1.5 kg. How long does this take?

(c) Find half-life time.

30. The population p of bacteria at time t is given by $p = 100e^{0.05t}$. Calculate

(a) the value of p when $t = 0$;

(b) How long will it take for the size of the population to double?

31. A machine was purchased for \$10000. Its value V after t years is given by $V = 10000e^{-0.3t}$. The machine must be replaced at the end of the year in which its value drops below \$1500. Determine in how many years the machine will need to be replaced.

32. The area A km 2affected by a forest fire at time t hours is given by $A = A_0e^{kt}$. When $t = 5$, the area affected is 1 km^2. Given that $A_0 = \frac{1}{e}$,

(a) Show that $k = 0.2$;
(b) Find the value of t when $100km^2$ are affected.

33. The population of a city at the end of 1972 was 250 000. The population increases by 1.3% per year.

(a) Write down the population at the end of 1973.

(b) Find the population at the end of 2002.

(c) How long will it take for the size of the population to exceed 400 000?

34. A population of bacteria is growing at the rate of 2.3% per minute. How long will it take for the size of the population to double? Give your answer to the nearest minute.

35. Each year for the past five years the population of a certain country has increased at a steady rate of 2.7% per annum. The present population is 15.2 million.

(a) What was the population one year ago?

(b) What was the population five years ago?

36. $1000 is invested at 15% per annum interest, compounded monthly. Calculate the minimum number of months required for the value of the investment to exceed $3000.

37. There were 1420 doctors working in a city on 1 January 1994. After n years the number of doctors, D, working in the city is given by $D = 1420 + 100n$.

(a) (i) How many doctors were there working in the city at the start of 2004?

(ii) In what year were there first more than 2000 doctors working in the city?

At the beginning of 1994 the city had a population of 1.2 million. After n

years, the population, P, of the city is given by

$P = 1\ 200\ 000\ (1.025)^n$.

(b) (i) Find the population P at the beginning of 2004.

(ii) Calculate the percentage growth in population between 1 January 1994 and 1 January 2004.

(iii) In what year will the population first become greater than 2 million?

(c) (i) What was the average number of people per doctor at the beginning of 1994?

(ii) After how many complete years will the number of people per doctor first fall below 600?

38. Initially a tank contains 10 000 litres of liquid. At the time t = 0 minutes a tap is opened, and liquid then flows out of the tank. The volume of liquid, V litres, which remains in the tank after t minutes is given by $V = 10\ 000\ (0.933)^t$.

(a) Find the value of V after 5 minutes.

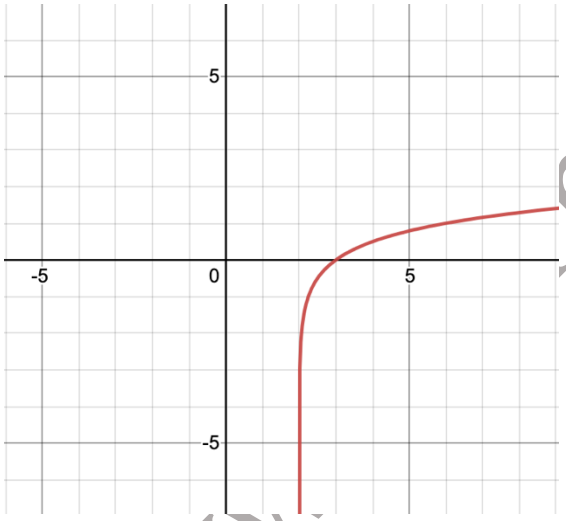
(b) Find how long, to the nearest second, it takes for half of the initial amount of liquid to flow out of the tank.

(c) The tank is regarded as effectively empty when 95% of the liquid has flowed out. Show that it takes almost three-quarters of an hour for this to happen.

(d) (i) Find the value of 10 000 – V when t = 0.001 minutes.

(ii) Hence or otherwise, estimate the initial flow rate of the liquid. Give your answer in litres per minute, correct to two significant figures.

ANSWER KEY

1. $3+\log_5 2$
2. 5
3. 31
4. 2
5. a+4b
6. 1.602
7. 6
8. 1
9. 3/2
10. (a+1)/(a+2)
11. $4e^2$.
12. 36
13. 3m/4
14. 

15. 1
16. 2
17. e/2
18. 9
19. (e+1)/(e-1)
20. 9/2
21. (6,10]
22. 2-2a
23. $\frac{1}{e^3}$, e^4
24. 9
25. 7
26. e^3
27. 1, e^9
28. a)22, b)5.76, c)0, d)1.73
29. a)4, b)4.90, c)3.47
30. a)100, b)ln2/0.05
31. 7
32. a)0.2, b)28
33. a)25300 b)368000
34. 30
35. a)14.8 million, b)13.3 million
36. 89 months
37. a)i)2420 ii)6
    b)i)1.54 million, ii)28%, iii)21
    c)i)845, ii)15 years
38. a)7070, b)10 minutes, c)3/4hours, d)690